THE CULTURAL IMPACT OF MATHEMATICS: UNIT I

CHAPTER 2 - The School of Athens by Raphael

Occasionally a work of art appears which so uniquely expresses the spirit of its time that it becomes by itself a symbol of that age. The Parthenon (448-432 B.C.), for example, is a reminder of the “Golden Age of Pericles," the summit of Greek civilization.

http://en.wikipedia.org/wiki/Parthenon

The final rebuilding of Chartres Cathedral (1194-1220 A.D.) stands as the quintessence of Gothic ideals.

The culmination of Byzantium is expressed in the Hagia Sophia of Istanbul (532-537 A.D.) or what was then called Constantinople.

Hagia Sophia - Wikipedia, the free encyclopedia

All of these works are on a monumental scale, as if to say, that if an age is to be captured it must be done in grand terms.

If one were to focus on the Renaissance period, however, and seek such a representative from the field of architecture, the search would be informative, replete with promising candidates, but in the end no single structure could stand in the same class as the examples given above.

St. Peter's (1546-1657 A.D.) might come to mind, but it was not completed until 100 years after the Renaissance period had expired, and it is more symbolic of the Counter-Reformation than anything else.

St. Peter's Basilica - Wikipedia, the free encyclopedia

Although there were many truly excellent and innovative works of architecture during the Renaissance, the honors for reflecting that age must go to the painters who raised their art to the same level of significance that had been hitherto reserved for architecture. If not St. Peter's, then most assuredly, Michelangelo's Sistine Ceiling (1508-1512 A. D.). For certainly this is a monument on the grandest scale by one of the greatest artists of all time.

http://en.wikipedia.org/wiki/Sistine_Chapel_ceiling

Again this choice would have to be rejected, for the Sistine frescoes transcend the time of their creation and reflect values which are so personal and at the same time, so universal, that they cannot be limited to a particular time or place .

We need only walk a few yards from the Sistine Chapel to find the work which fulfills our requirements. It is also a fresco, entitled The School of Athens (1510-1511) by the great Italian artist, Raphael (1483-1520) .
http://en.wikipedia.org/wiki/School_of_Athens
http://en.wikipedia.org/wiki/File:Sanzio_01.jpg

In the papal apartments of the Vatican, it occupies one wall of a room known as the Stanza dellaSegnatura (Room of the Signing of Papal Letters). This is the first room in a series painted by Raphael after he was summoned to Rome in 1508 by Pope Julius II. The fresco is one of four adorning the room, all of which are devoted to the areas of learning: theology, philosophy, law, and the arts. The School of Athens celebrates the study of philosophy and commemorates its origins by portraying the great philosophers of Ancient Greece. As Janson [1] says, "... The School of Athens has long been acknowledged as Raphael's masterpiece and the perfect embodiment of the classical spirit of the High Renaissance."
(Refer to the following link for a description of some of the individuals in the fresco):
http://en.wikipedia.org/wiki/File:Raffaello_Scuola_di_Atene_numbered.svg

The two central figures striding forth out of the past are Plato (14), on the left, and Aristotle (15), on the right; the two greatest figures of Ancient Greek philosophy that were known to Renaissance scholars.

Around them are distributed the philosophers, mathematicians, musical theorists, and grammarians who were linked historically with the ideas of Plato and Aristotle. All of them are involved in active discourse, study, or silent contemplation, revealing the unselfconscious passion for disinterested inquiry that is the central core of true philosophy. Some of the more famous are easy to identify. To the left of Plato, Socrates (12) is engaged in discussion with a small group. In the lower left foreground, Pythagoras (6), known as the Father of Mathematics, holds a book in which he is inscribing the ideas from the slate held before him. On the lower right, Euclid (18) or possibly, Archimedes, wields a compass demonstrating a geometric proof.

Above him, and to the right, is Ptolemy (20) with his back to us, the great astronomer, holding a globe representing his geocentric system of the universe. He is wearing a crown because Renaissance scholars confused him with Ptolemy I, who became king of Egypt after Alexander the Great died. Raphael was fond of using his friends and associates as models for his figures. Zoroaster (19) also holding a celestial globe, has the features of Leonardo da Vinci, and the bent over figure of Euclid belongs to his architect friend, Bramante. In fact, the background of the entire painting is taken from Bramante's plans for the rebuilding of St. Peter's, a work he would never live long enough to execute.

Just to the right of Zoroaster peering at the viewer, is Raphael himself (R). The fact that he chose to be a conspicuous part of this illustrious gathering is something we shall conjecture about later.

To continue Diogenes (13), famous for his legendary search for an honest man, lies sprawled on the steps, relaxed yet intent on the manuscript he is reading. An interesting story is told about Diogenes that is consistent with his portrayal in this fresco. As his fame spread throughout Greece, Alexander the Great came to pay homage to him. So impressed was the great conqueror with this man sworn to poverty that he asked him if he had any wish, and it would be granted. Whereupon Diogenes answered, "Yes, stand out of my light."

The pensive philosopher, Heraclitus (13), is positioned at front center and his presence is generally considered Raphael's tribute to

Michelangelo. The figure is a derivative of those sculptured giants Michelangelo had painted on the Sistine Ceiling. The story has it that one day when Michelangelo was away, Raphael had an opportunity to peek at the Sistine frescoes then under construction. Michelangelo was working on his masterpiece at the same time that The School of Athens was being completed.

Why is The School of Athens 'acknowledged as Raphael's masterpiece and the perfect embodiment of the classical spirit of the High Renaissance' ? Kenneth Clark says, "Raphael was a man of his age. He absorbed and combined all that was being felt or thought by the finest spirits of his time. He is the supreme harmonizer ... The thoughts to which he gave visible expression in the papal apartments were a synthesis as all inclusive as the summaries of the great medieval theologians."[2]

Indeed, it has been said of the Renaissance that its greatest philosophers were its artists. Men like Brunelleschi, Alberti , da Vinci, and Piero della Francesca thought of art as a discipline that could stand on the same level as the other liberal arts and sciences. All of them were theorists and practitioners in their art. They relied heavily on mathematical foundations to organize their compositions and their writings served as training manuals to the artists of their time.

Their works reveal a revival of interest in the classical spirit of Ancient Greece and Rome. They were fortunate in having access, through the new medium of printing, to Latin translations of the ancient texts. Many of these texts were preserved through a thousand years of neglect in Western Europe, thanks to the efforts of scholars from Byzantium and Islam. This great restoration commenced in the Middle Ages and reached its culmination in the 16th century at the height of the Renaissance.

The political and social climate of Italy in the 15th century produced a group of creative individuals who were more than ready to exploit this regained knowledge. Raphael was a direct heir to this tradition and his knowledge informs every detail of The School of Athens. The presence of Raphael and his contemporary artistic brethren (da Vinci, Bramante, Michelangelo) in the painting is understandable. They conceived themselves as descendants of this philosophic tradition, believers in the Platonic ideal that the only life worth living is that which can be examined.

The School of Athens clearly indicates that Renaissance philosophy. Where Aristotle's philosophy had occupied stage center during the late Middle Ages, formally reconciled in Christian theology by St. Thomas Aquinas, he is now presented literally on an equal footing with his mentor, Plato. The humanist scholars of the Renaissance had commenced to examine both of these men's ideas with a fresh perspective untrammeled by the involutions of medieval theological dogma.

A case in point is the book that Plato carries in the fresco. It is one of the most important philosophic works in the history of western ideas. Called the Timaeus (Latin: Timeo), it is a summary of the Pythagorean cosmology as refined by Plato's original philosophic thought. In the dialogue written by Plato, Timaeus. an astronomer and mathematician, explains the origins of the universe, its microcosmic and macro-cosmic structure, and the existence of a creator to bring it all about. Although the emphasis is theological, stressing the image of God the Geometer, as Designer of the World, it also contains within it the seeds of a scientific explanation for the structure of matter and space.

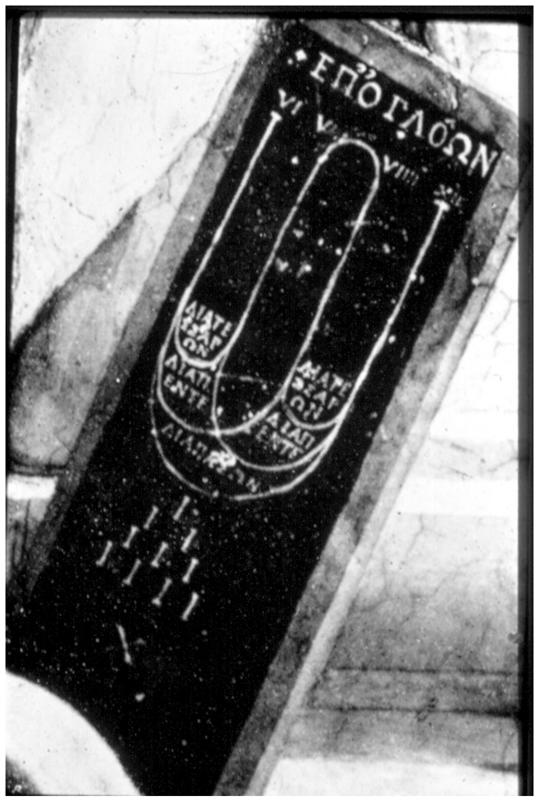
http://en.wikipedia.org/wiki/File:God_the_Geometer.jpg

For a thousand years it had served the fathers of the Church as a foundation for building the doctrines of the Faith. From the Renaissance on, it would provide an inspiration to those who would examine the world of nature in order to discover the great design attributed to the Creator.

The mathematical content of the Timeus provides us with some important clues concerning the composition of Tho School of Athens.

To begin, note the activity of Pythagoras at the left foreground of the fresco.

An enlargement of the slate held before him reveals its secrets.



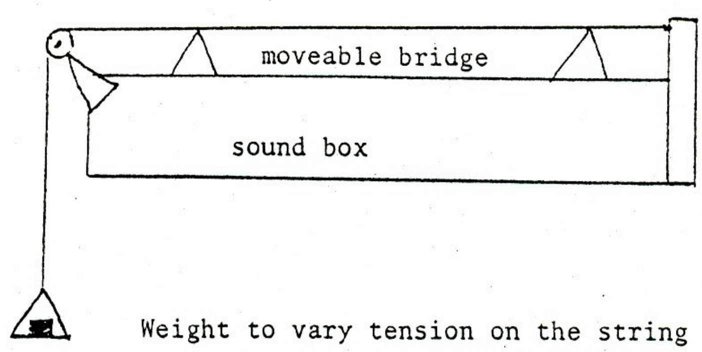
A network links the numbers 6,8, 9, and 12 written in Roman numerals from left to right. The words contained in the loops are the names for the musical intervals determined by the ratios of the numbers connected by the network. Thus, the ratio of 6 to 8 or 3:4 is the diatesseron. The ratio of 6 to 9 or 2:3 is called the diapente. The lowest loop relates 6 to 12 or 1:2 and is the diapason. Moving upward and to the right, another diapente is given by 8 to 12, and the last interval, 9 to 12, is again the diatesseron. In modern musical terms, the diatesseron is called "the perfect fourth," the diapente is called the "perfect fifth," and the diapason is called the "octave."

This connection of the most harmonious of the musical intervals with numerical ratios is acknowledged as the first step in the scientific derivation of the laws of nature. It served as the foundation of the Pythagorean belief that mathematics could unlock the secrets of nature.

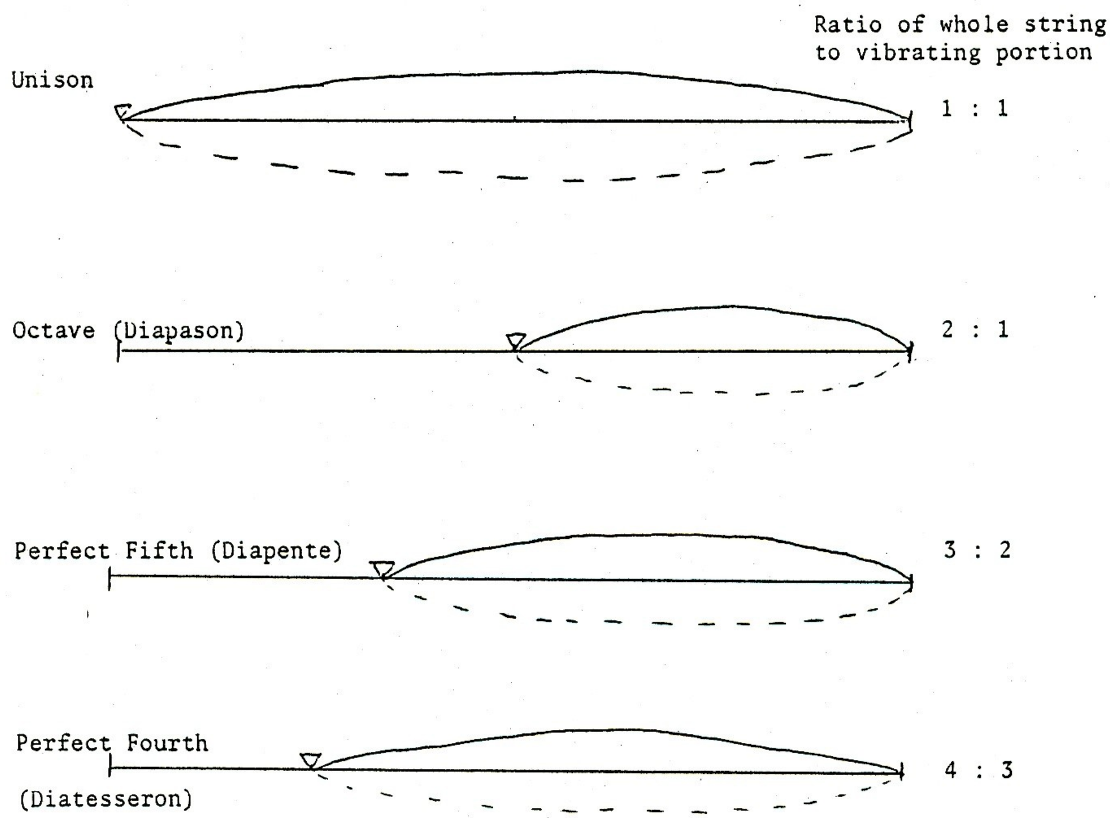
To clarify the origin of these ratios, refer to the figure below. The mono- chord was a device that Pythagoras was reputed to have used for demonstrating the musical intervals and their relationship to numerical ratios.

A stretched string is attached at opposite ends of a resonating box and given sufficient tension to produce an audible tone called the "fundamental" when plucked or bowed. When depressed at the midpoint, the two halves will each produce a tone having twice the frequency of the fundamental. These new tones are both an octave higher than the original (the diapason). The next figure shows that if the string is depressed so that it is divided .into one-third and two-thirds of its length, the one-third portion will produce a tone three times the frequency of the fundamental and the two-thirds portion will vibrate at 3/2 times the fundamental. Note how 1/3 the length is related to 3 times the frequency as a reciprocal relationship. This is also true of 2/3 the length and 3/2 times the frequency. The tone

The Pythagorean monochord. Instrument to demonstrate the musical ratios.



String lengths correlated to the tones produced.



sounded as 3/2 times the frequency is the perfect fifth (the diapente). It is the next most harmonious interval after the octave. Also, it follows that the tone produced by 1/3 of the string length is an octave higher than the perfect fifth, since multiplying the frequency of a tone by 2 produces its octave, i.e. 3 = 2 x 3/2.

In a similar way the interval of the perfect fourth can be derived. If the string is divided into 1/4 and 3/4 of its length, the portion 3/4 in length will vibrate in a frequency that is 4/3 of the fundamental. This tone is called the perfect fourth (the diatesseron) and was considered the next most harmonious interval after the perfect fifth. The other portion of the string, 1/4 of its length, produces a tone that sounds two octaves higher than the fundamental or 4 times its frequency.

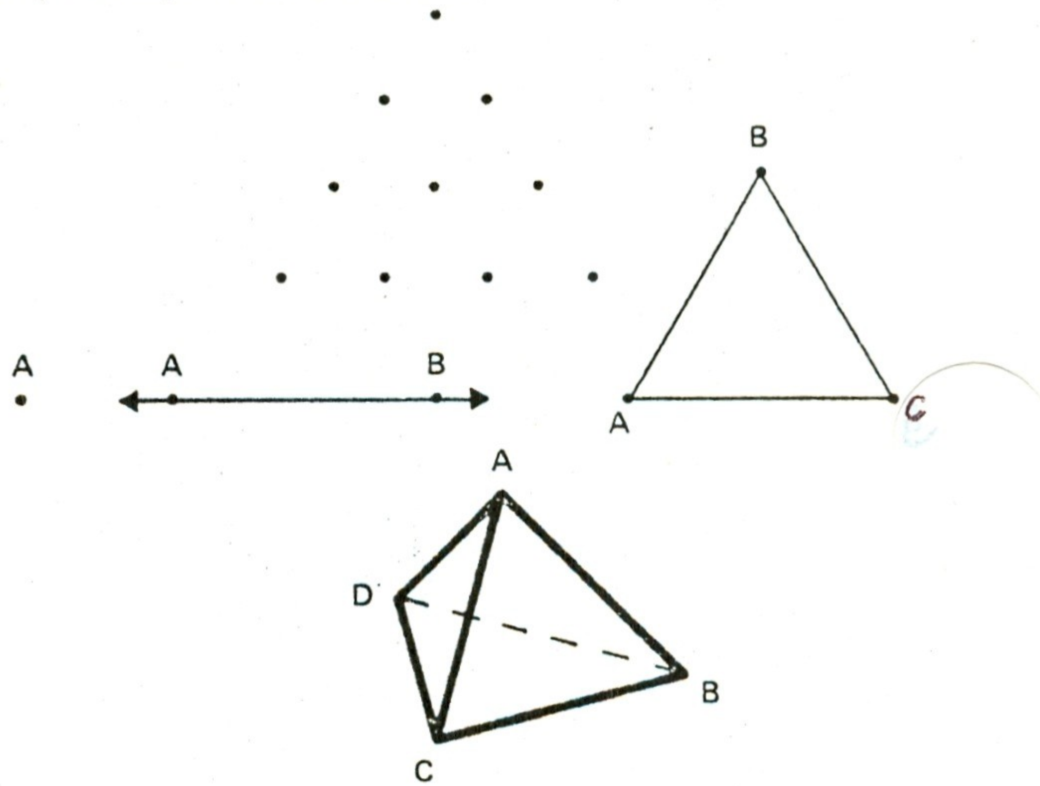
The modern names for the intervals are most easily explained by looking at their relative positions in a notated scale built upon the fundamental. The figure below shows such a scale built upon a fundamental tone at middle C (the C major scale). Tones are shown in ascending order of their frequencies, the lines and spaces filled in order as the scale is ascended. The names and ratios have been placed with each tone. The interval ratios refer to comparing the frequencies of the tones to that of the fundamental. The derivation of the ratios given will be covered in detail in the unit on Mathematics and Music.

The C Major Scale with names and ratios of tones related to a fundamental.

For the present the analysis of the slate before Pythagoras will be continued. Below the diagram of the musical ratios is the triangular number 10, its Roman numeral completes the slate. In Pythagorean philosophy the number 10 is of special significance. It is known as "the holy tetraktys" and it symbolizes the number of perfection and the totality of the universe. http://en.wikipedia.org/wiki/Tetraktys

As Boyer notes,[3] in his History of Mathematics: it included "the sum of all possible geometric dimensions. A single point is the generator of dimensions, two points determine a line of dimension one, three points (not on a line) determine a triangle with a dimension of two, and four points (not in a plane) determine a tetrahedron with volume of dimension three; the sum of the numbers representing all dimensions therefore is the revered number ten. It is a tribute to the abstraction of Pythagorean mathematics that the veneration of the number ten evidently was not dictated by anatomy of the human hand or foot."

The "Holy Tetraktys" as the Sum of All Dimensions



The geometric representation of the number 10 that is depicted in The School of Athens is typical of the Pythagorean conception of numbers in general. The sequence of numbers described by Boyer above yields yet another sequence which results from adding the points at each successive level, namely: 1,3,6,10. This sequence could be extended indefinitely and is called the sequence of triangular numbers.

The number ten, the holy tetraktys, was embodied in the earliest non-geocentric astronomical system developed by the Pythagorean, Philolaus fl. 410 B.C.). In his system, the Earth, Sun, and Moon were considered as planets along with Mercury, Venus, Mars, Jupiter, and Saturn (the outer planets, Uranus, Neptune, and Pluto hadn't been discovered). All of these revolved about a Central Fire (not the Sun). The stars were considered as a single entity moving about the Central Fire in opposite motion to the Earth, thereby explaining the observed diurnal motion of the night sky. To fill out the complement of ten objects, a Counter Earth was hypothesized, ever invisible to earthlings and located on the opposite side of the Central Fire. Philolaus' astronomical theory may have been an attempt to create a synthesis between the holy tetraktys and the so called "harmony of the spheres". This is a probable explanation for the linking of the musical ratios and the holy tetraktys on the slate held before Pythagoras in The School of Athens. The early Pythagoreans connected the musical ratios with the motions of the heavenly spheres, believing the planets each sounded a particular tone consistent with the ratios involved in their movements. Fanciful as this idea may seem to us from our modern viewpoint, it persisted as a strongly held belief down to the dawn of the scientific revolution in the 17th century.

In his fascinating study of 'Pythagorean Cosmology and Renaissance Poetics' entitled, Touches of Sweet Harmony[4] S. K. Heninger, Jr. provides a moving analysis of Shakespeare's involvement with this theme. He quotes from the famous lines of The Merchant of Venice:

How sweet the moonlight sleeps upon this bank!
Here will we sit and let the sounds of music
Creep in our ears; soft stillness and the night
Become the touches of sweet harmony.
Sit, Jessica. Look how the floor of heaven
Is thick inlaid with patines of bright gold;
There's not the smallest orb which thou behold'st
But in his motion like an angel sings,
Still quiring to the young-ey'd cherubins.
Such harmony is in immortal souls,
But whilst this muddy vesture of decay
Doth grossly close it in, we cannot hear it.

A few short years after Shakespeare wrote these immortal lines, Johannes Kepler published his book entitled Harmonices Mundi (1619), (Translated 'The Harmony of the World').

As Koestler has written,[5]:"Shakespeare's lines could well serve as the motto for Kepler's book." In this" ... all-embracing synthesis of geometry, music, astrology, astronomy, and epistemology ... ", Kepler is mainly concerned with finding melodic scale structures for the planets based upon their velocities and distances from the Sun. Tucked away in this highly mystical work is his third great law of planetary motions. This law accurately relates the periods of revolution of the planets about the Sun with their average distance from the Sun.

In the fifteenth century, it was the writings of Alberti (1404-72), a brilliant humanist, which were responsible for reactivating the Pythagorean theory of proportions, declaring it to be the foundation of beauty in all of the arts. Using music as an example, he extended the mathematical principles of harmony to the domain of spatial and human proportions. In this respect, he was hardly being original. The sculptors and architects of antiquity and the Middle Ages were well practiced in the use of proportions and Alberti was an avid observer of their works that still existed in the Italy of his time.

Like most humanists, his writings paid homage to the authority of this magnificent heritage. But his main contribution in this respect was the aesthetic connection he made between the simple ratios of music and the art of visual representation. As he says in his treatise De Re Aedificatoria,[6] " ... I conclude that the same numbers, by means of which the agreement of sounds affects our ears with delight, are the very same which please our eyes and our mind."

The artists of the Renaissance seized upon this dictum and became the true heirs of the Pythagorean tradition. Everything in their visual expression became subject to the rules of proportion; the composition of the human body, modular plans for designs , the dimensions for architecture , and the planning of cities.

http://en.wikipedia.org/wiki/File:Da_Vinci_Vitruve_Luc_Viatour.jpg

All of these examples implied an absolute standard of perfection which coincided with the harmonious view of the universe. Number relationships could unlock the secrets of creation and bring order out of chaos. The microcosmic order of music was unified with the macro-cosmic plan for the movements of the planets and stars.

The School of Athens is a magnificent reflection of this conception. In all of its parts it is a visual choreography of the Pythagorean musical cosmos. The slate on the left tells us the numerical ratios that are to be used

(1:2, 2:3, 3:4) and the slate over which Euclid-Bramante hovers with his compasses, yields up the secret of how these ratios are realized in the visual organization of Raphael's composition.
http://en.wikipedia.org/wiki/File:Sanzio_01_Euclid.jpg

Fresco painting is an art that requires utmost dexterity and careful planning. The method requires the application of watercolors to fresh wet plaster. As the plaster dries, a chemical reaction produces a film which covers the watercolors, making them an inherent part of the wall's surface. The artist must work quickly and accurately. Since ancient times it was the practice to prepare a grid on the surface of the wall which subdivided it into manageable areas. A “cartoon" was made for each subregion, much like a simple jigsaw puzzle. The cartoon was applied on the wet plaster and the outlines of each figure were indented with a pointed implement. The cartoon was then removed and with the aid of a sketch containing the color schemes, the colors were applied.

We know that Raphael worked from a full-size drawing for by a miracle, it still survives. The precise dimensions of the cartoon grid can only be surmised. We can speculate, however, that it was incorporated into the total structural framework that shall be described below. No claim is here being made that the analysis provided by the author is that which was used by Raphael in the execution of his masterpiece. It is based upon the clues that Raphael integrated within the scene (the Pythagorean slate, the Euclidean construction) and the strictly defined disposition of the figures and architecture. The evidence is overwhelming that such a framework guided the distribution of elements in the composition. It provided the following objectives: 1) a precise handling of focused perspective; 2) the grouping and modeling of figures; 3) the arrangement of architectural elements; 4) balance and symmetry of parts; 5) the relationships between groups of figures; 6) a guide for the viewer to perceive and absorb the details of the composition; 7) the creation of a harmonized totality.

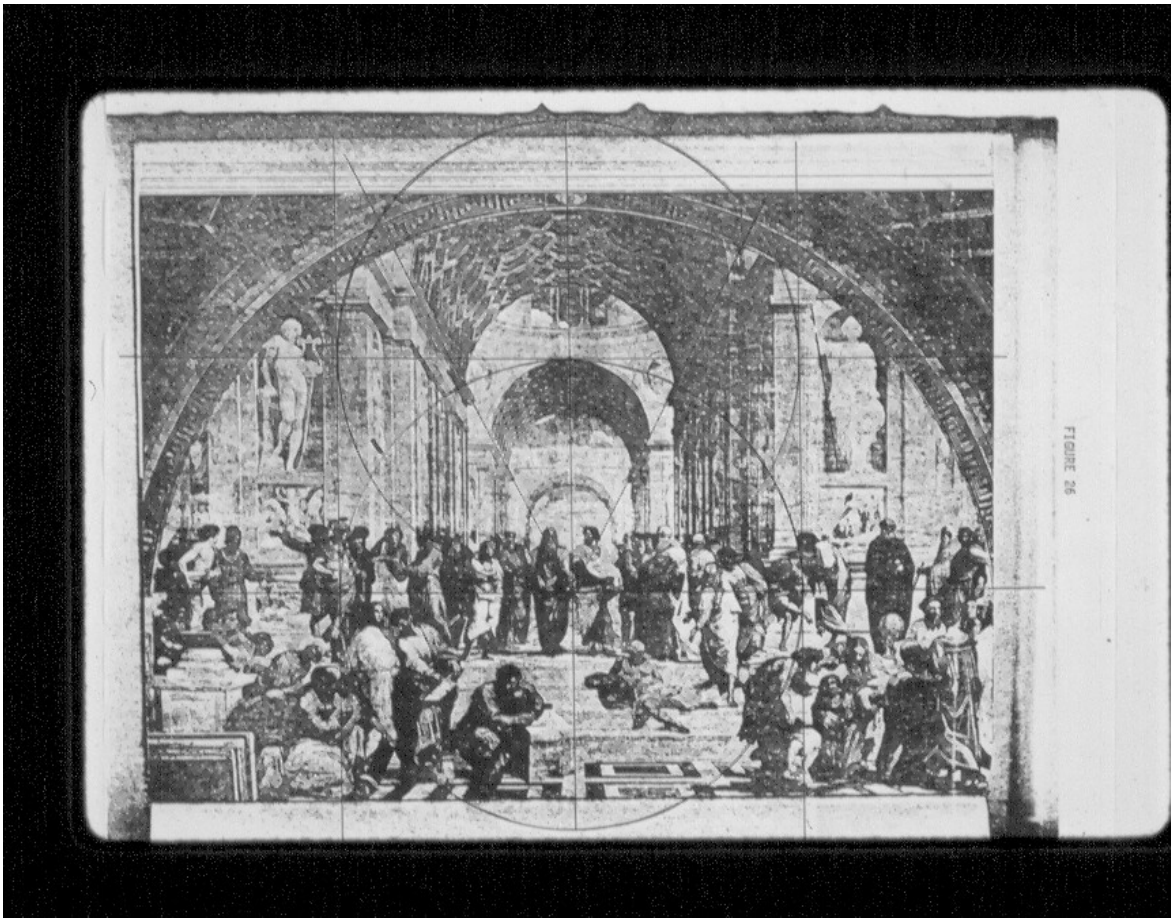
The first point to be emphasized is that whether we take the measure of the fresco itself or work from a reduced scale print of it, the basic proportions to be found in The School of Athens are essentially preserved despite distortions that are inevitable in reproduction. Our attention will be directed initially to the overall design.

With the fresco before us we become aware immediately of its bilateral symmetry. To locate the central axis of symmetry, lines of perspective may be drawn along the bases of the arches. Their intersection is the vanishing point, located between Plato and Aristotle. (Figure 26 below)

The central axis may be drawn passing through the center of the circular ornament at the top of the arch framing the fresco, through the vanishing point and along the corner of the pedestal on which Heraclitus leans.

The basic unit for dimensioning the entire fresco is given by the distance from the vanishing point to the apex of the middle arch. Using this length as a radius a central circle has been drawn. With the same radius and center at the apex of the arch, another circle is drawn above the first. These two overlapping circles yield the height of the fresco and its border.

Next, by construction, perpendicular lines are drawn tangent to the ends of the horizontal central axis of the circle that passes through the vanishing point. Again, the basic radius is laid off and perpendicular lines connected to divide the horizontal into four congruent regions. It may now be seen that the fresco is contained in a rectangle that is four units long and three units high. It is clear that the musical ratio of 4:3, the perfect fourth, determines its overall geometry.



Having deduced the general dimensions of the fresco, we now turn our attention to its internal structure. The vertical axis of symmetry divides it into the octave ratio of I:2. This separates the Plato-Pythagorean group on the left and the Aristotelian-Euclidean group on the right. In Figure 27 we can see another realization of the octave ratio in the progression of barrel vaults structured from rear to front, their successive diameters at the base also in the 1:2 ratio. If another vault were constructed in the foreground, it would terminate with Pythagoras at the left base and Euclid's demonstration on the right.

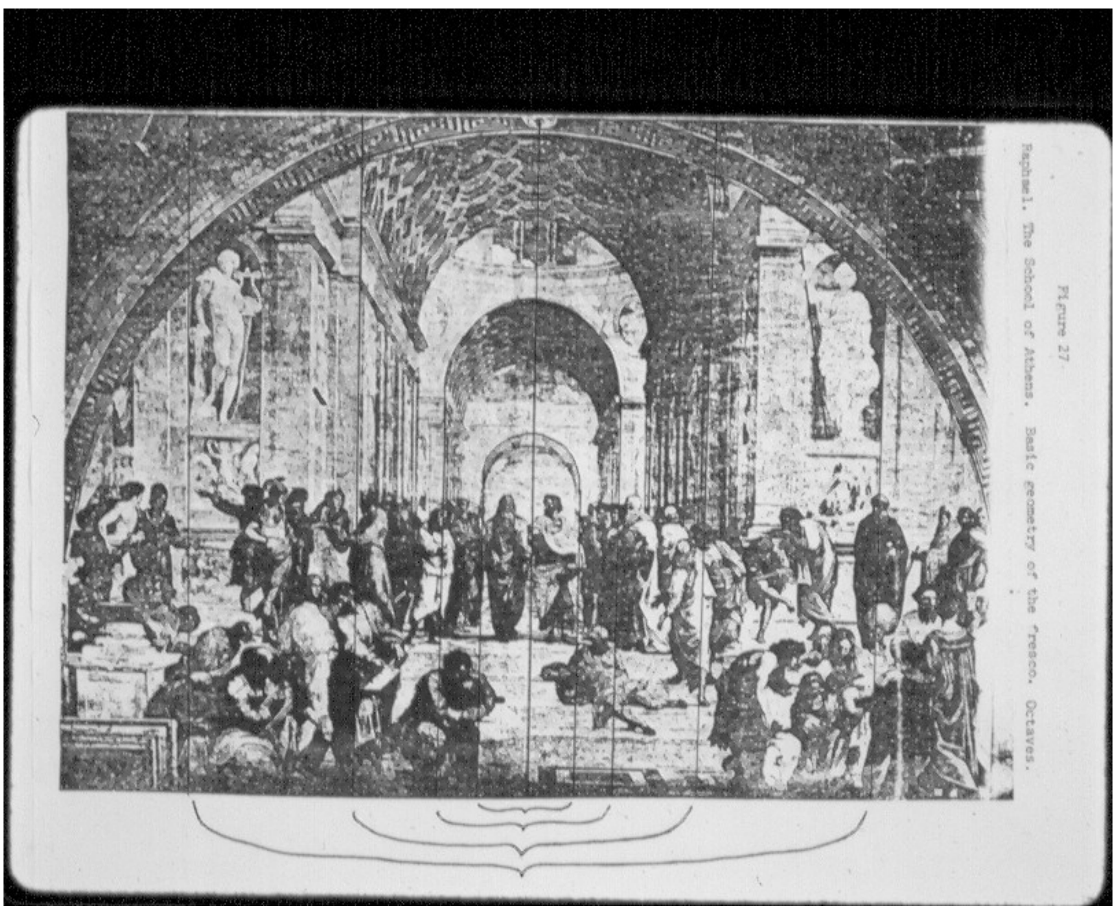


Figure 27
Raphael. The School of Athens. Basic geometry of the fresco. Octaves.

Furthermore, if the fresco is divided into eight equal parts along the horizontal, the eight vertical bands give us a complete octave of a modal scale, the basis of music in the Renaissance. These have been numbered from left to right in Figure 28 for identification.

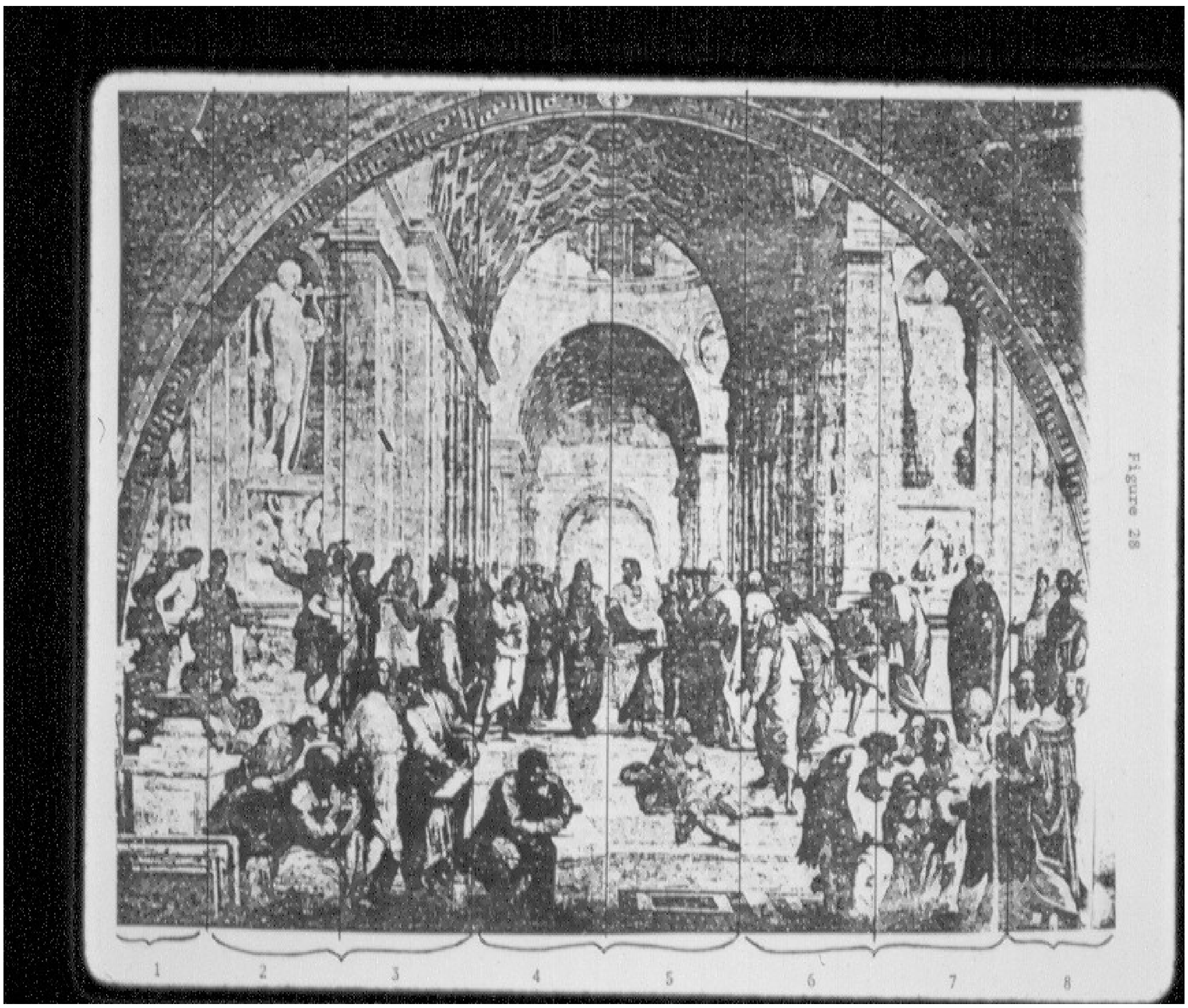


Figure 28

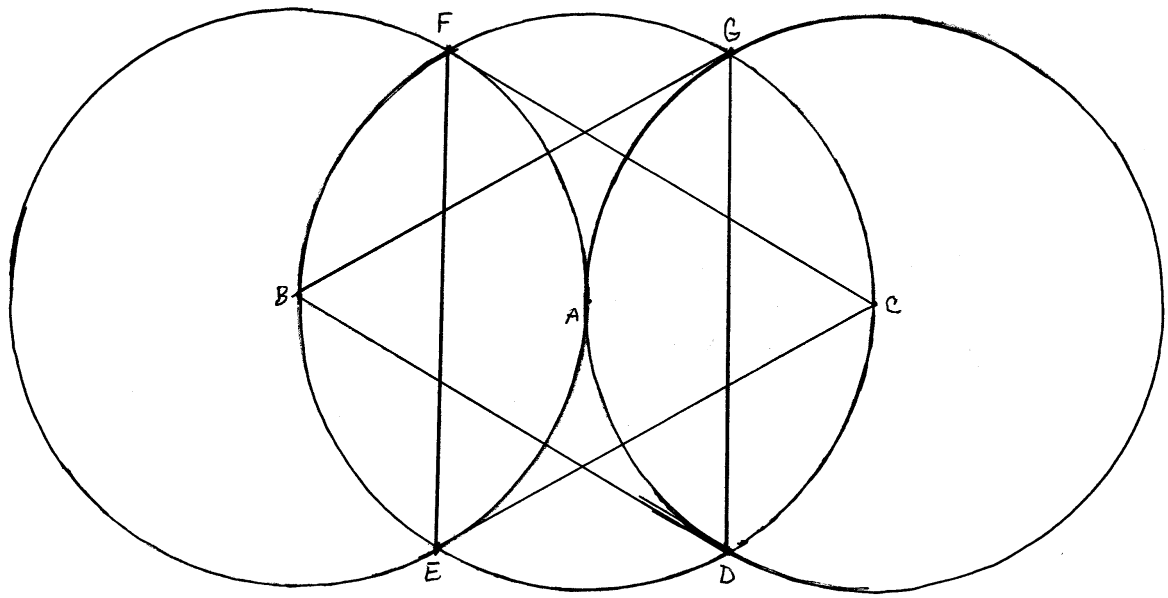
Radiating from the center, bands 4 and 5 enclose the self-contained group abound Plato and Aristotle and also include the foreground figures of Heraclitus and Diogenes. Bands 2 and 3 contain the statue of Apollo, the group in discussion with Socrates, and in the foreground, the group gathered around Pythagoras. Bands 6 and 7 contain the statue of Athena, the pair of figures mounting the stairs, the boys studying under the statue, the solitary figure to their right, and the group around Euclid in the fore-ground. Bands 1 and 8 include the side groups that complete the fresco. Thus, it is by this 8-part symmetry that Raphael has vertically organized his figures into self-contained groups.

Before we examine the internal organization further, let us prepare the setting for this framework by returning to the contents of the fresco. The book which Aristotle holds is his Ethics. In it he says that art, or applied science (as it was once defined), is the capacity or trained ability of making things according 'to a true rule.'[7]

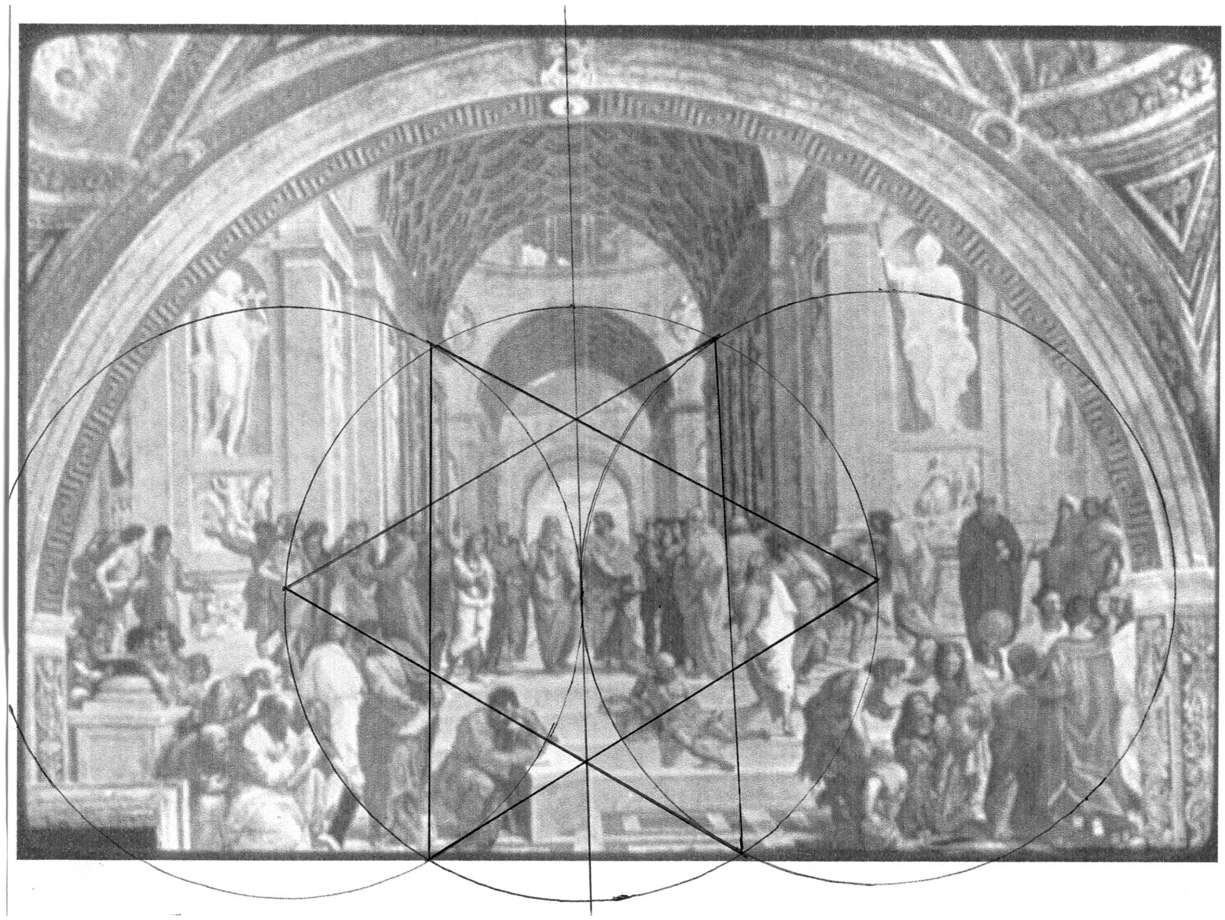
The rules to be found in Renaissance art are suggested by the bent over figure of Euclid- Bramante absorbed in the construction on the slate before him. As we shall see, it is the construction on the slate that largely determines the internal geometry of the fresco.

http://en.wikipedia.org/wiki/File:Sanzio_01_Euclid.jpg

It may be questioned why Raphael selected his close friend, Bramante, as the model for Euclid. Bramante was a famous Renaissance architect, responsible for the first set of plans for the proposed reconstruc-tion of St. Peter's. Raphael adopted some of his ideas in the architectural setting of The School of Athens. In addition, it is known that Bramante was also the author of a treatise (still to be discovered) on the theory of proportions. He had the proper credentials, therefore, for his preeminent role in the fresco. If we now turn our attention to the details of the figure on Euclid's slate, we shall see how well Raphael learned his lessons from Bramante. The figure is that of a star hexagon with internal lines drawn to form a contained parallelogram with a diagonal also indicated. Euclid has just completed drawing an arc across the star point farthest away from him. Is it pure speculation that this construction has anything to do with the composition of the fresco? This writer is of the opinion that Raphael was too much of a craftsman to place a meaningless figure on that slate. Coupled with his belief in Pythagorean cosmology, it seems reasonable that he duplicated the slate's microcosm in the macrocosm of the entire fresco. (Figure 29) shows the construction.



To begin, a circle is drawn. Two additional circles with the same radius pass through the center of the first circle resulting in 3 intersecting circles. The centers are labeled A, B, and C. Next, using a straight edge, lines are drawn from the centers of the outside circles to the intersections of the opposite circles with the circle A. Finally, lines are drawn connecting the intersections of each outside circle with the central circle A. The 6 star points are labeled as B though G. (Figure 30) shows the framework superimposed on the fresco.

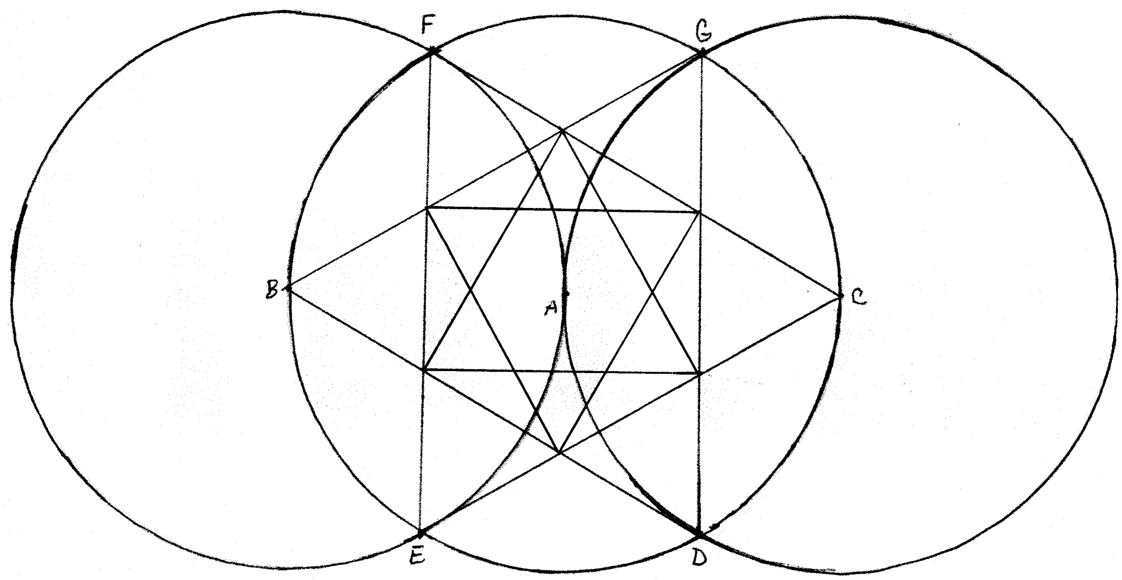


. The fact the Euclid is wielding a pair of compasses implies that the circle has a critical role to play in the fresco's structure. Raphael was particularly fond of the circular framework, as his marvelous tondo paintings will attest. With the artists of his time, a single opening of the compass was preferred. This has already been supplied for our analysis in the basic division of the fresco described in Figure 26. Now, in combination with the hexagonal framework, which provides center points for constructing circular arcs, the circular framework is displayed in Figure 30. The static quality of the hexagon is modified to conform to the curving flow of the circular arcs. Not only do the arcs separate and relate the groups in foreground, background, left and right, but they also contribute to regulating the design of individual figures in their sculptured attitudes and gestures.

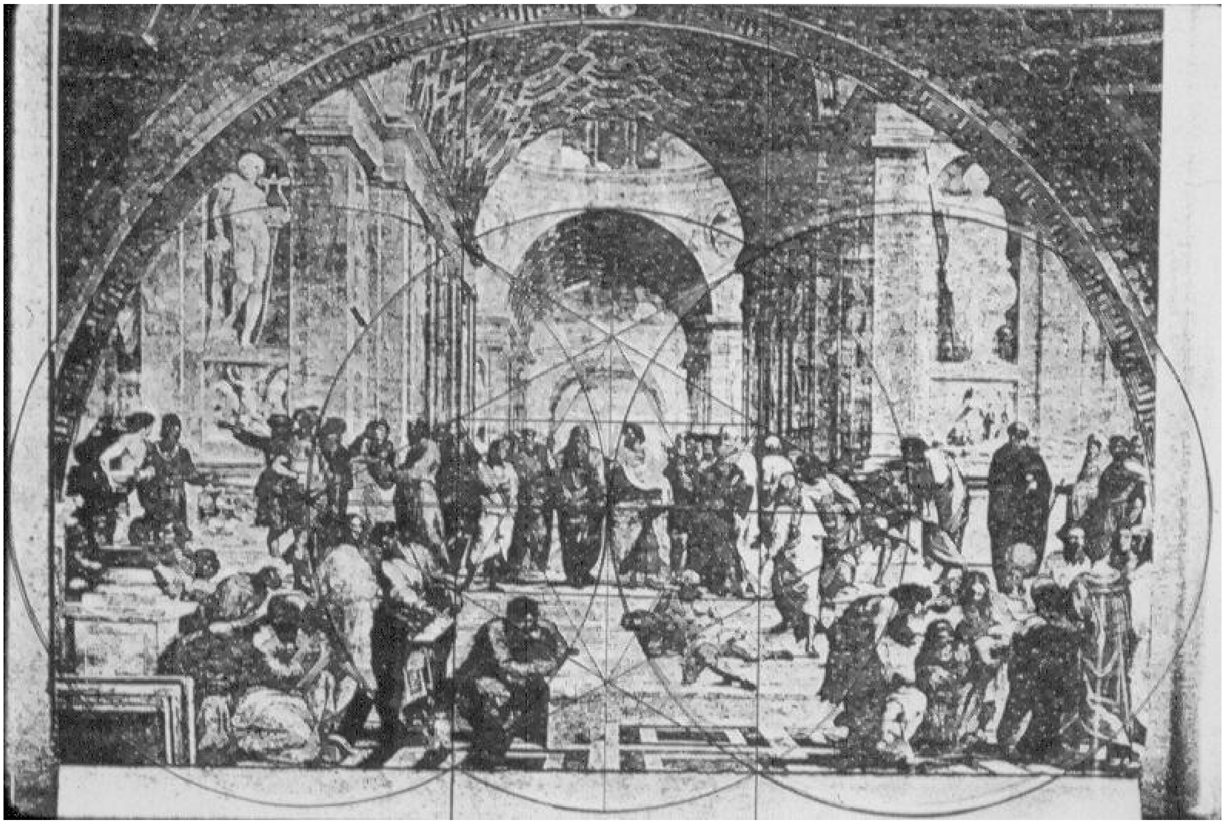
First, and foremost, the outside circles completely separate the Plato and Aristotle halves of the fresco, focusing attention on the larger groupings of individuals. The central circle also acts to enclose the important figures of that section. Notice how the lune formed by the left and central circles frame around Plato to include Heraclitus, Hypatia, and Socrates. The lune on the right wraps around Aristotle, Diogenes and the couple ascending the stairs. What is remarkable is how the curvature of the lunes emphasize the bodily gestures of the participants.

Next, the star hexagon can be seen in Figure 29 to contain a central hexagon. In Figure 30 this narrows the focus to contain all of the central figures connected to Plato and Aristotle, as well as the top portions of Heraclitus and Diogenes, whose forms are completed by the star point appendages linked to the hexagon. As an additional help to the organization of individuals, note how the line from B to D in Figure 29, links the positions of the heads of Hypatia, the figure holding a copy book, and Heraclitus. Also, the line from E to C in Figure 29, links the lower extremities of Heraclitus, Diogenes, the couple ascending the stairs, and the boy studying at the top of the stairs.

To complete the use of Euclid's construction, (Figure 31) shows the insertion of a smaller star hexagon inscribed within the central hexagon.



When superimposed upon the fresco in (Figure 32), it serves to encompass the height and width of the central figures.



At this point the reader may wish to study the circular and hexagonal frameworks to see how the details of the composition conform to its structure. Was it actually used by Raphael to position, relate, and model the content? The author is convinced that this framework, or something close to it, guided the artist in organizing and rendering the complex details of the scene. Admittedly, there is no documentation for this assertion.

However, there is much evidence of a general kind that this kind of approach was a common approach of artistic craftsmanship during the Renaissance period. It is reflected in the belief that Raphael was the "supreme harmonizer" of his time.

Returning for a last look at The School of Athens we shall focus again on Plato and Aristotle, the two central protagonists.
http://en.wikipedia.org/wiki/File:Sanzio_01_Plato_Aristotle.jpg
Their characteristic gestures give us a strong hint about the content of their argument. Aristotle's main criticism of Plato revolves about his theory of knowledge. In particular, it engages upon their opposed concepts about universals. We will not go into detailing this great debate that still continues to our present day except to note how each man's gesture symbolizes his point of view. Plato points toward heaven to the realm where universals have objective existence. Of course, this is not heaven in the Christian sense but the mind of God, the only place where the eternally unchanging can exist. This, to Plato, is the only reality. As he says in the Timaeus[8] "We must in my opinion begin by distinguishing between that which always is and never becomes from that which is always becoming but never is. The one is apprehensible by intelligence with the aid of reasoning, being eternally the same, the other is the object of opinion and irrational sensation, coming to be and ceasing to be, but never fully real."

He then goes on to prove by this argument that the world, which we know through our senses, cannot be real, but only a model built out of the patterns and forms which exist as abstractions in the mind of the Creator.

Aristotle points ahead of himself, not to an abstract heaven, but to the world about us, suggesting that reality consists only of individual and specific objects. Sets or classes of objects are only handy abstractions that allow us to order our environment.

As Durant says[9]: "Aristotle ... is a realist in the modern sense; he is resolved to concern himself with the objective present, while Plato is absorbed in a subjective future."

Plato, with his mathematical bias, is concerned with theories and generalities, while Aristotle, the biologist, directs his attention to flesh and blood realities, the world as it exists to the senses. On the other hand, Durant's dichotomy is too restrictive, for we find in Aristotle's metaphysics that "Aristotle ... agrees with Plato that the universal is real. If it were not, there could be no knowledge of anything. The world is knowable precisely because everything possesses an intelligible structure, which, as its form, places it in its appropriate class. Aristotle insists, however, that this form does not exist apart from the individual. The universal, qua universal does not exist in the objective world. What does exist are individual objects which possess a common essence or form.” [10]

The distinctions between Plato's and Aristotle's arguments about how we know things were fundamental to the development of science as well as art. Aristotle's theory of statements[11] and his interests in Biology were to lead to a firm foundation for the classification of living organisms.

In retrospect, he may be considered the father of descriptive science which contributed immensely to the ordering of natural phenomena. The emphasis he placed on the logical structure of statements had a strong influence on Euclid's formulation of the Elements. His ideas in this area would form the basis for deductive argument down to the present day. Plato, in contrast, placed tremendous importance on mathematics as a stepping-stone leading to an understanding of his theory of universals.

He says of mathematicians in The Republic: “The diagrams they draw and the models they make are actual things ... but now they serve in their turn as images, while the student is seeking to behold those realities ,which only thought can apprehend.”[12]

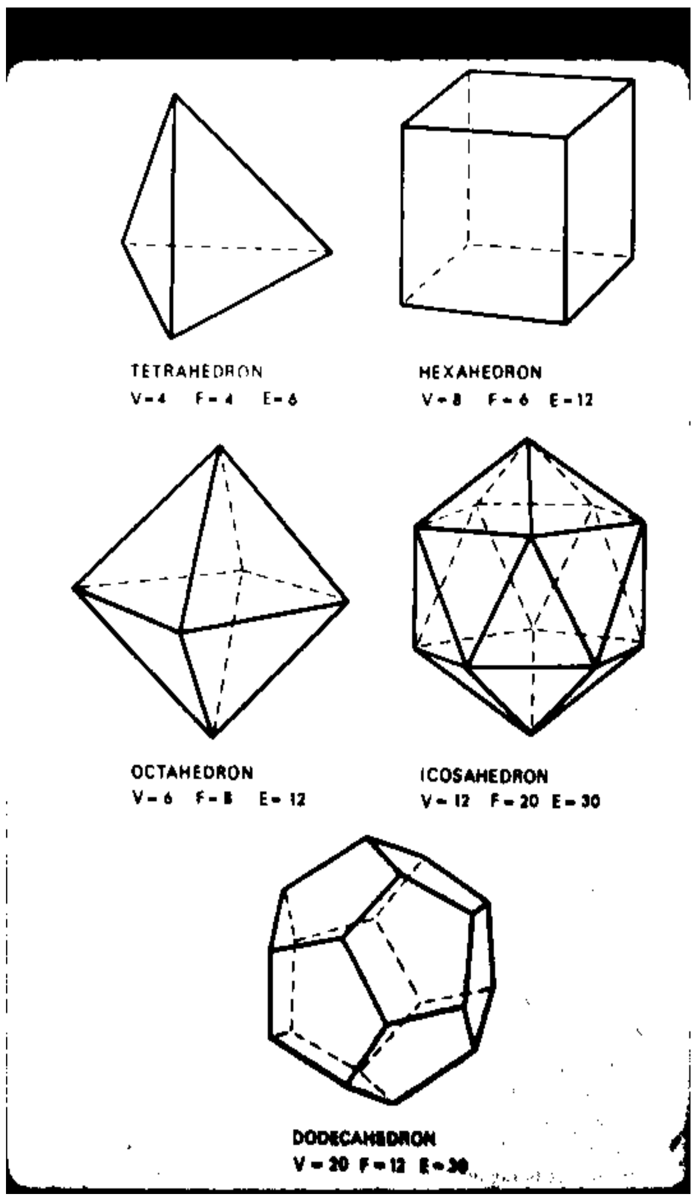
Thus, for example, the visual proof of the Pythagorean Theorem is merely a sensory aid to an abstraction that can only exist in the mind. The finest drawing can only approximate the dimensions that truly image the generality.

http://en.wikipedia.org/wiki/Pythagorean_theorem

Plato's placement of mathematics in the sequence of cognitive stages to philosophical knowledge was to have more profound impact on the subsequent evolution of science than Aristotle's descriptive approach.

As Cornford reflects in his translation of the passage above: "Conversely, the fact that the mathematicians can use visible objects as illustrations indicates that the realities and truths of mathematics are embodied, though imperfectly, in the world of visible and tangible things ... “[13]

In the Timaeus, for example, Plato plants the seed for a mathematical atomic theory of matter by relating the regular solids to the “elements" as they were known in his time (Figure 33). By clever reasoning through analogies, he concludes that fire is tetrahedral in form, air is octahedral, water is icosahedral, and earth takes its form from the cube. Plato was somewhat vague about the assignment of the dodecahedron. He says: "There still remained a fifth construction, which the god used for arranging the constellations on the whole heaven."[14]



After making these assignments he makes the following remarkable and prophetic statement; "We must, of course, think of the individual units of all four bodies as being far too small to be visible, and only becoming visible when massed together in large numbers; and we must assume that the god duly adjusted the proportions between their numbers, their movements, and their other qualities and brought them in every way to the exactest perfection permitted by the willing consent of necessity."[15]

Though vague, imprecise, and incorrect in its details, Plato's theory of matter is one of the first attempts to provide a mathematical description of nature. It is this quantitative, as opposed to the qualitative Aristotelian approach, that would become the foundation of scientific method. One need only glance through a modern textbook of Chemistry to feel the spirit of Plato hovering over our continuing efforts to understand the basic structure of matter.

And yet, the 'irrefutable fact' still remains that Raphael placed himself und his artistic brethren in the Aristotelian sector of The School of Athens. Perhaps it was this passage in Aristotle's Poetics which dictated his choice: "Artistic creation ... springs from the formative impulse and the craving for emotional expression. Essentially, the form of art is an imitation of reality; it holds the mirror up to nature. There is in man a pleasure in imitation, apparently missing in lower animals. Yet the aim of art is to represent not the outward appearance of things, but their inward significance; for this, and not the external mannerism and detail, is their reality .... The noblest art appeals to the intellect as well as to the feelings ... and this intellectual pleasure is the highest form of joy to which a man can rise., Hence a work of art should aim at form, and above all at unity, which is the backbone of structure and the focus of form."[16]

Enough evidence has been presented to draw the conclusion that The School of Athens is a comprehensive blend of art, philosophy, and mathematics. Even more significant is what it tells us about the Renaissance period itself. The painting generates a feeling of supreme confidence and passionate enthusiasm about artistic creativity. Never has man, "the rational animal" been depicted with more respect and optimism about his potential. The statues of Apollo and Athena, the Olympian Gods of light and reason, reminds us that this was a unique moment in the history of mankind. Despite the darkness and ignorance that was to fall over man's dominion in the wake of the religious wars to follow, a seed had been planted here that would emerge and blossom in the philosophy of the 17th century. In a most prophetic way, The School of Athens announced the birth of modern civilization.

LIST OF REFERENCES

Chapter 2 - The School of Athens

1. Janson, History of Art, New York: Prentice-Hall, 1963, p. 370

2. Clark, Civilisation, New York: Harper and Row, 1969, p. 129

3. Boyer, History of Mathematics, New York: Wiley, 1968, p. 58f

4. Henninger, Touches of Sweet Harmony, San Marino, Calif.: Huntington, 1974, p. 4-5

5. Koestler, The Watershed, Garden City, N.Y.: Doubleday, 1960, p. 214

6. Quoted in Bouleau, op. cit., p. 85 (see note on 2 -14)

7. Jancor, The Philosophy of Aristotle, New York: Monarch, 1966, p. 173

8. Plato, Timaeus, tr. by Lee, Baltimore: Penguin, 1971, p. 40

9. Durant, The Story of Philosophy, New York: Pocket Library, 1954, p. 60

10. Jancor, op. cit., p. 116

11. Bunt, Jones, Bedient, The Historical Roots of Mathematics, Prentice-Hall, 1976, p. 133-137

12. Plato, The Republic, tr. Cornford, New York: Oxford University Press, 1955, p. 225

13. Ibid.

14. Plato, Timaeus, Ope cit., p. 75

15. Ibid., p. 78

16. Durant, op. cit., p. 73